Calculus Foundations — Lab Report

Differentiation and Integration for ENGR/Stats

Objective: Provide a rigorous yet friendly refresher on derivatives and integrals, including core rules, intuition, and worked examples for immediate course use.

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Objective and Theory

Objective

Review the essential calculus tools for this course: derivatives (rates of change), integrals (accumulation/area), and two key techniques (substitution, parts).

Notation

Derivative: dy/dx, $f'(x)$, $d/dx\, f(x)$. Integral: $\int f(x)\, dx$. Definite: $\int_a^b f(x)\, dx$.

Differentiation — core rules

$\frac{d}{dx}x^n = nx^{n-1}$, $\frac{d}{dx}(c) = 0$, $\frac{d}{dx}(cf) = cf'$

$\frac{d}{dx}(f+g) = f' + g'$, Product: $(uv)' = u'v + uv'$, Quotient: $(u/v)' = (u'v - uv')/v^2$

Chain: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Common: $\frac{d}{dx}e^{ax} = ae^{ax}$, $\frac{d}{dx}\ln x = 1/x$ (for $x > 0$), $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$

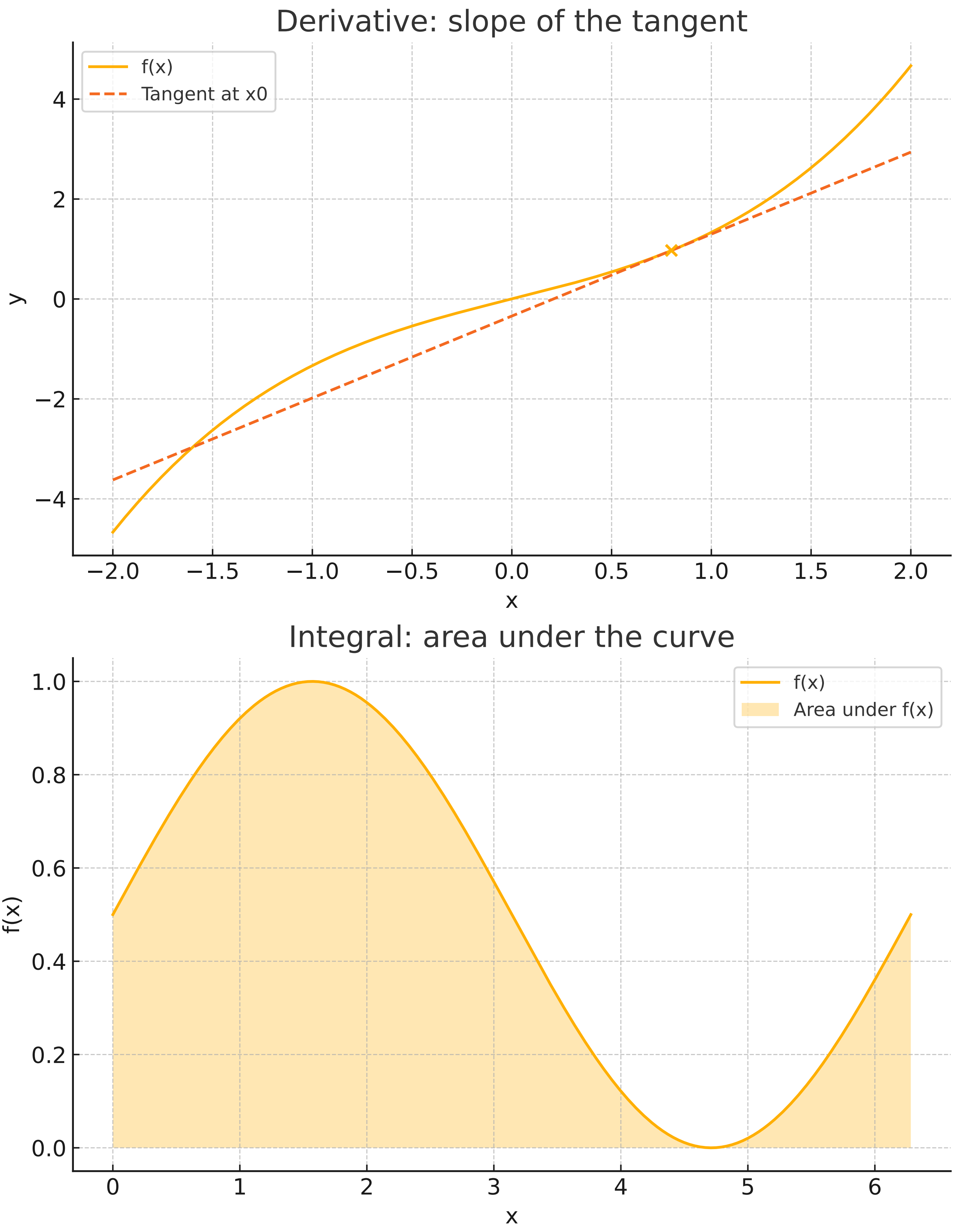
Integration — core rules

Find F with $F' = f$; then $\int f(x)\, dx = F(x) + C$.

$\int x^n dx = x^{n+1}/(n+1) + C$ ($n \neq -1$), $\int c\, dx = cx + C$, linearity holds.

$\int e^{ax} dx = (1/a)e^{ax} + C$ ($a \neq 0$).

Visual Intuition



Core Techniques

u-substitution (reverse chain rule)

Let $u = g(x)$, $du = g'(x)\,dx$. Then $\int f(g(x))g'(x)dx = \int f(u)\,du$.

Example: $\int 2x\,e^{x^2}dx$, take $u = x^2 \Rightarrow du = 2x\,dx$: $\int e^u du = e^u + C = e^{x^2} + C$.

Integration by parts

From product rule: $d(uv)/dx = u'v + uv' \Rightarrow \int u\,dv = uv - \int v\,du$.

Choose u to simplify when differentiated; choose dv easy to integrate.

Example: $\int ye^{-ky}dy$ $(k > 0)$. Set $u = y$, $dv = e^{-ky}dy$

$\Rightarrow du = dy$, $v = -e^{-ky}/k$. Then $\int ye^{-ky}dy = -ye^{-ky}/k - e^{-ky}/k^2 + C$.

Improper Integrals and Worked Examples

Improper integrals (used often in probability)

For $a > 0$: $\int_0^\infty e^{-ax} dx = 1/a$, $\int_b^\infty e^{-ax} dx = e^{-ab}/a$.

Also: $\int_b^\infty x e^{-ax} dx = e^{-ab}(b/a + 1/a^2)$.

Differentiation examples

$\frac{d}{dx}(3x^4 + 2x^2 - 5) = 12x^3 + 4x$

Product rule: $\frac{d}{dx}(x^2 e^{2x}) = (2x + 2x^2)e^{2x}$

Integration examples

$\int_0^1 x^2 dx = [x^3/3]_0^1 = 1/3$

By parts: $\int x e^{2x} dx = (x e^{2x})/2 - (e^{2x})/4 + C$

Results, Discussion, and Quick Reference

Results & Discussion

You should now be able to compute core derivatives and integrals, set up definite integrals, and use substitution and integration by parts. The figures show geometric meaning: derivative = slope of tangent, integral = area under the curve.

Conclusion

Key formulas and ideas to remember are summarized below. Keep this page handy as a cheat sheet.

Quick Reference

$\frac{d}{dx}x^n = nx^{n-1}, \quad \int x^n dx = x^{n+1}/(n+1) + C \ (n \neq -1)$

$\frac{d}{dx}e^{ax} = ae^{ax}, \quad \int e^{ax} dx = (1/a)e^{ax} + C$

$\frac{d}{dx}\ln x = 1/x \ (x > 0), \quad \int 1/x \, dx = \ln|x| + C$

Integration by parts: $\int u \, dv = uv - \int v \, du$

u-substitution: with $u = g(x)$, $\int f(g)g' dx = \int f(u) du$