**ENGR 200 — Chapter 4 Notes (4.1–4.4) — Q&A Solutions**

**Q1. Define a discrete random variable (give 2 examples).**

**Answer:**

* A discrete random variable takes values in a countable set (finite or countably infinite).
* Examples: number of defects on a part; number of emails received in an hour.

**Q2. Define a continuous random variable (give 2 examples).**

**Answer:**

* A continuous random variable takes values over an interval (uncountably many possibilities).
* Examples: time to failure; temperature at noon.

**Q3. PMF vs PDF: State what each represents and the normalization rule.**

**Answer:**

* PMF (discrete): p(x) = P(X = x), with Σ p(x) = 1.
* PDF (continuous): f(x) ≥ 0 and the total area is 1: ∫ f(x) dx = 1. Probabilities are areas: P(a < X ≤ b) = ∫ₐᵇ f(x) dx.

**Q4. State the two core properties a valid PDF f(x) must satisfy.**

**Answer:**

* Nonnegativity: f(x) ≥ 0 for all x.
* Normalization: ∫ f(x) dx over the support equals 1.

**Q5. Sketch a possible PDF and label axes (area under curve = 1).**

**Answer:**

* Any nonnegative curve whose total area is 1 (students’ sketches vary).
* Example: a triangle on [0,2] with f(x)=x/2 for 0≤x≤2 and 0 otherwise.

**Q6. Definition: Write F(x) in terms of X.**

**Answer:**

* F(x) = P(X ≤ x).

**Q7. List three properties of any CDF (monotonicity, limits, continuity).**

**Answer:**

* Nondecreasing (monotone).
* Right-continuous.
* Limits: lim x→−∞ F(x) = 0 and lim x→+∞ F(x) = 1.

**Q8. Relationship: If F is differentiable, relate f and F.**

**Answer:**

* If F is differentiable at x, then f(x) = dF(x)/dx.

**Q9. Compute P(a < X ≤ b) in terms of F.**

**Answer:**

* P(a < X ≤ b) = F(b) − F(a).

**Q10. Write the formula for the mean μ = E[X] in terms of f(x).**

**Answer:**

* μ = E[X] = ∫ x f(x) dx (integrate over the support of X).

**Q11. Write the formula for E[g(X)] and for E[X^2].**

**Answer:**

* E[g(X)] = ∫ g(x) f(x) dx (over the support of X).
* E[X²] = ∫ x² f(x) dx.

**Q12. Write the formula for variance Var(X) using E[X^2] and μ.**

**Answer:**

* Var(X) = E[X²] − (E[X])².

**Q13. Describe in words what Uniform(a,b) models; draw a sketch.**

**Answer:**

* Uniform(a,b) models a value equally likely anywhere between a and b (flat density).
* Sketch: a horizontal line on [a,b] at height 1/(b−a).

**Q14. Write the PDF f(x).**

**Answer:**

* f(x) = 1/(b−a) for a ≤ x ≤ b; 0 otherwise.

**Q15. Write the CDF F(x) (piecewise).**

**Answer:**

* F(x) = 0 for x < a; (x−a)/(b−a) for a ≤ x ≤ b; 1 for x > b.

**Q16. Mean and Variance for Uniform(a,b).**

**Answer:**

* Mean μ = (a+b)/2 ; Variance Var(X) = (b−a)² / 12.

**Q17. Probability example: For X~U(a,b), compute P(a ≤ X ≤ c) for a≤c≤b.**

**Answer:**

* For X ~ U(a,b) and a ≤ c ≤ b: P(a ≤ X ≤ c) = (c − a) / (b − a).