**ENGR 200 — Chapter 4 Notes (****4.1–4.4, 9/24/2025) — Write-in Worksheet**

Continuous vs. Discrete • PDFs • CDFs • Mean • Variance • PDF↔CDF • Uniform(a,b)

**4.1 Discrete vs. Continuous Random Variables**

Q1. Define a discrete random variable (give 2 examples).

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Q2. Define a continuous random variable (give 2 examples).

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Q3. PMF vs PDF: State what each represents and the normalization rule.

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**4.1 Probability Density Function (PDF) — General Form**

Q4. State the two core properties a valid PDF f(x) must satisfy.

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Q5. Sketch a possible PDF and label axes (area under curve = 1).

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**4.2 Cumulative Distribution Function (CDF) — General Form**

Q6. Definition: Write F(x) in terms of X.

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Q7. List three properties of any CDF (monotonicity, limits, continuity).

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Q8. Relationship: If F is differentiable, relate f and F.

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Q9. Compute P(a < X ≤ b) in terms of F.

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**4.3 Mean and Variance — General Forms (Continuous)**

Q10. Write the formula for the mean μ = E[X] in terms of f(x).

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Q11. Write the formula for E[g(X)] and for E[X^2].

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Q12. Write the formula for variance Var(X) using E[X^2] and μ.

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**4.4 Uniform Distribution U(a,b)**

Q13. Describe in words what Uniform(a,b) models; draw a sketch.

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Q14. Write the PDF f(x).

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Q15. Write the CDF F(x) (piecewise).

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Q16. Mean and Variance for Uniform(a,b).

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Q17. Probability example: For X~U(a,b), compute P(a ≤ X ≤ c) for a≤c≤b.

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