Term Project Requirement: Efficient Frontier Analysis

FIN435, Spring 2024

The Efficient Frontier Analysis term project is aimed at providing students with an in-depth understanding of portfolio management and asset allocation strategies. Students will not only enhance their understanding of portfolio management concepts but also develop analytical and decision-making skills applicable to real-world investment scenarios.

**Objective:**

* The primary objective of this project is to enable students to comprehend the principles of portfolio management by analyzing the trade-offs between risk and return through the concept of the efficient frontier.

**Learning Outcome:**

By completing this project, students will:

* Develop a solid understanding of asset allocation and its role in portfolio construction.
* Gain insights into the risk-return trade-offs inherent in investment decisions.
* Learn to use Excel analytical tools to identify optimal portfolio compositions based on risk preferences and return objectives.
* Enhance their skills in financial analysis and decision-making within the context of investment management.

**Project Components:**

* Data Collection: Gather historical price data for a selection of stocks from a reliable financial data source such as Yahoo Finance.
* Data Preparation: Clean and organize the data, focusing on the "date" and "adjusted close" columns, and combine the data for multiple stocks into a single Excel file.
* Analysis: Calculate key metrics including average return, standard deviation (risk), correlation matrix, and covariance matrix for the selected stocks.
* Efficient Frontier Construction: Plot the efficient frontier graphically, showcasing various combinations of risk and return for different portfolio allocations.
* Optimization: Use optimization techniques such as Solver in Excel to determine the minimum portfolio standard deviation for each target return level along the efficient frontier.

**Report:**

* Prepare a comprehensive report documenting the data collection process, analysis methodology, findings, and insights gained from the efficient frontier analysis.
* Please include the Excel spreadsheet containing your analysis results, including calculations and the graph of the efficient frontier, along with your report submission.

**Project Guidelines:**

* Adhere to academic integrity principles throughout the project, ensuring proper citation of data sources and methodologies.
* Seek guidance from instructors when facing challenges or uncertainties during the project.

**Assessment Criteria:**

* Accuracy and completeness of data collection and analysis.
* Clarity and depth of insights gained from the efficient frontier analysis.
* Demonstrated understanding of portfolio management principles and risk-return trade-offs.
* Effectiveness of communication in presenting findings and conclusions.
* Adherence to project guidelines and academic integrity standards.

**Appendix 1 – Steps**

In this project, you will obtain the efficient frontier for eight securities, such as the following:

1. Cadbury Schweppes Public (CSG)
2. Home Depot (HD)
3. Citigroup (C)
4. Southwest Airlines (LUV)
5. Texas Instruments (TXN)
6. Johnson and Johnson (JNJ)
7. IBM (IBM)
8. Boeing (BA)

First, you need to obtain closing prices at the end of each month for these securities. To obtain these closing prices, go to the following web site and follow the appropriate links: [www.yahoo.com](http://www.yahoo.com)

Select “Finance”. Once this page is loaded, enter the symbol code (letters in parentheses following the name of the company) for one of the firms listed above in the “Enter Symbol(s)” box at the top and then hit go. Once the page for that company is loaded, in the panel on the left-hand side, you will see a choice for “Historical Prices”. Select that link. Once you are on that page, you are asked for the dates for which you want to obtain the historical prices and with what frequency you want the data to be reported. Your start date is January 31, 2019 and your end date is January 31, 2024. Choose monthly observation interval. Scroll down on that page until you see “download to spreadsheet” option. When you select that option you are asked whether you want to open or save or cancel. Choose save and tell the program where you want the data to be saved and under what file name. Delete the rows containing dividend information. The only data you need from these excel files are the date and the adjusted closing prices. You can delete the rest of it. Notice that the data is sorted in descending order of date. Using these adjusted closing prices, calculate the monthly returns. You need to use the following formula to calculate these returns:



Since your data is in descending order, you have to be careful about which cell has Adj. Pricet and which cell has Adj. Pricet-1. Do this for all the companies listed above. Then gather the data in a single excel spreadsheet. Make sure that you label your columns to indicate the company for which you have the data stored in that column. Also, make sure that your dates matched for all the firms. You will not have a return figure for October 2007. So, your return data will start from November 2007 and runs till October 2012. You should have a total of 60 observations for each company. Your excel worksheet should look similar to the table below:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Date | CSG | HD | C | LUV | TXN | JNJ | IBM | BA |
| Jan 2019 |  |  |  |  |  |  |  |  |
| Feb 2019 |  |  |  |  |  |  |  |  |
| ... |  |  |  |  |  |  |  |  |

Calculate the average monthly return and standard deviation of this monthly return for all companies using the build in functions of excel. Now we need to annualize average monthly return and monthly standard deviation. You will use the following formulas to do this:

Average annual return = (1+Average monthly return)12 – 1

Annual standard deviation = Monthly standard deviation \* 

In excel, go to the “Tools” menu and see if you have an option called “Data Analysis”. If you do, you can skip the rest of the things in this paragraph. If not, you have to follow them. Go to the “Tools” and select “Add-Ins”. You will get a list of functions that you can add in. Select “Analysis ToolPak”, “Analysis ToolPak – VBA” and “Solver Add-In”. You will need the functions that are contained in these modules to do your assignment. If this adding-in process worked properly, you should see “Data Analysis” as an option when you select “Tools” menu.

Using correlation function in Data Analysis, calculate the pairwise correlation coefficients between our eight securities. You are going to have an 8\*8 matrix. Have the results reported in a separate sheet.

Using the relationship between correlation and covariance, calculate the covariance matrix for our securities:

Correlationij = Covarianceij/(St. Devi\*St. Devj) and

Covarianceij= Correlationij\*St. Devi\*St. Devj

Then calculate the bordered covariance matrix for an equally weighted portfolio of eight securities. You will invest 1/8th of your wealth in each of these company shares. Weights will be 1/8 = 0.125. Every cell in the bordered covariance matrix is the corresponding cell in the covariance matrix multiplied by the appropriate weights. For example, if I am trying to calculate the bordered covariance between security C and JNJ, then I’ll take the covariance of these securities from the covariance matrix and multiply it by weights of C and JNJ in the portfolio. For an equally weighted portfolio, this will be covarianceC,JNJ\*(0.125)\*(0.125). The variance of this equally weighted portfolio is equal to the sum of all points in the bordered covariance matrix. Calculate the expected return of the portfolio as well. At each of these steps, you should be entering formulas in appropriate cells instead of numbers themselves. So that when you change something in the worksheet, everything will be recalculated.

Now, you are ready to calculate the bordered covariance matrix for target returns. This is the one we are going to use to find our efficient frontier portfolios. This time, you don’t know the weights that you are going to assign to each of these securities. These weights will be the solutions that you obtain from “Excel Solver”. Go to the “Tools” menu and select “Solver”. In the “Target Cell”, enter the cell address of variable that you are trying to minimize or maximize. We are trying to minimize the standard deviation of the portfolio in this exercise subject to some constraints on weights of securities and the expected return of the portfolio. So, enter the address of the cell that contains the portfolio standard deviation formula in your worksheet in the target cell. In the “By Changing Cells”box, enter the cell addresses of the portfolio weights that you want the solver to find out for you. In “subject to constraints” cell, write down the constraints we are going to have on the weights and the portfolio expected return, one by one.

Problem we are solving can be formulized in the following manner:

Minimize σP

Subject to:

* 0 <= wi for all security i (we are going to have 8 of these constraints)
* wi<=1 for all security i (we are going to have 8 of these constraints)
* Σwi = 1
* E(RP) = X

Start with a portfolio return (X) little over the minimum annual return for our eight securities. Let’s say the minimum annual return for our securities is 9.78%. Then you can start with a portfolio expected return of 10%. After entering the constraints, select “Solve”. Excel solver is going to give you a set of weights that minimize the standard deviation of the portfolio subject to your constraints. Make sure you copy the portfolio expected return, portfolio standard deviation and the weights associated with this portfolio to the solution section of the worksheet. Change the expected portfolio return by a constant amount, and rerun the solver. Copy the necessary information to the solution section. Repeat this exercise until you reach the highest return for all eight securities. Make sure that you repeat these steps for at least 10 times.

Remember that we assumed the lowest return for our eight securities to be 9.78%. Therefore, we started with a portfolio expected return of 10%. Suppose that the highest return for our eight securities is 44.6 %. So, we will stop at 44%. If I would like to move from 10% to 44% in at least 10 iterations, every time I rerun the solver, I should increment the portfolio expected return by 3.4%. So, I’ll have solutions for 10%, 13.4%, 16.8% and so on so forth.

Last thing we need to do is to graph these portfolio expected returns and standard deviations.

**Appendix 2 - Formulas**

Portfolio Return:

**Portfolio Return = w1\*r1 + w2\*r2 + w3\*r3 + w4\*r4 + w5\*r5 + w6\*r6 + w7\*r7 + w8\*r8**

where: ***w1, w2, w3, w4, w5, w6, w7, w8*** are the weights of each stock in the portfolio, and ***r1, r2, r3, r4, r5, r6, r7, r8***are the returns of each stock in the portfolio.

Portfolio Standard Deviation:

**Portfolio Standard Deviation = sqrt(w12\*σ12+ w22\*σ22+ w32\*σ32 + w42\*σ42+ w52\*σ52+ w62\*σ62 + w72\*σ72+ w82\*σ82 + 2\*w1\*w2\*ρ12\*σ1\*σ2 + 2\*w1\*w3\*ρ13\*σ1\*σ3 + 2\*w1\*w4\*ρ14\*σ1\*σ4 + 2\*w1\*w5\*ρ15\*σ1\*σ5 + 2\*w1\*w6\*ρ16\*σ1\*σ6 + 2\*w1\*w7\*ρ17\*σ1\*σ7 + 2\*w1\*w8\*ρ18\*σ1\*σ8 + 2\*w2\*w3\*ρ23\*σ2\*σ3 + 2\*w2\*w4\*ρ24\*σ2\*σ4 + 2\*w2\*w5\*ρ25\*σ2\*σ5 + 2\*w2\*w6\*ρ26\*σ2\*σ6 + 2\*w2\*w7\*ρ27\*σ2\*σ7 + 2\*w2\*w8\*ρ28\*σ2\*σ8 + 2\*w3\*w4\*ρ34\*σ3\*σ4 + 2\*w3\*w5\*ρ35\*σ3σ5 + 2\*w3\*w6\*ρ36\*σ3\*σ6 + 2\*w3\*w7\*ρ37\*σ3\*σ7 + 2\*w3\*w8\*ρ38\*σ3\*σ8 + 2\*w4\*w5\*ρ45\*σ4σ5 + 2\*w4\*w6\*ρ46\*σ4\*σ6 + 2\*w4\*w7\*ρ47\*σ4\*σ7 + 2\*w4\*w8\*ρ48\*σ4\*σ8 + 2\*w5\*w6\*ρ56\*σ5\*σ6 + 2\*w5\*w7\*ρ57\*σ5\*σ7 + 2\*w5\*w8\*ρ58\*σ5\*σ8 + 2\*w6\*w7\*ρ67\*σ6\*σ7 + 2\*w6\*w8\*ρ68\*σ6\*σ8 + 2\*w7\*w8\*ρ78\*σ7\*σ8 )**

where: **σ1, σ2, σ3, σ4, σ5, σ6, σ7, σ8** are the standard deviations of each stock in the portfolio. **ρ12, ρ13, ρ14, ρ15, ρ16, ρ17, ρ18, ρ23, ρ24, ρ25, ρ26, ρ27, ρ28, ρ34, ρ35, ρ36, ρ37, ρ38, ρ45, ρ46,ρ75, ρ48, ρ56, ρ57, ρ58, ρ67, ρ68, ρ78** are correlation coefficients between the stock returns. They represent the pairwise correlations between the stocks in the portfolio.

For example, **ρ12** represents the correlation coefficient between the returns of stock 1 and stock 2, **ρ23** represents the correlation coefficient between the returns of stock 2 and stock.

**Appendix 3 - About the CML (Capital market line, optional)**

To draw a tangent line from the risk-free rate to the efficient frontier, follow these steps:

* **Determine the risk-free rate:** The risk-free rate is the rate of return an investor can earn with zero risk. It is typically represented by the yield on a short-term U.S. Treasury bill.
* **Find the portfolio with the highest Sharpe ratio:** The Sharpe ratio is a measure of risk-adjusted return that takes into account the portfolio's expected return and standard deviation. The portfolio with the highest Sharpe ratio is the portfolio that offers the best risk-adjusted return.
* **Calculate the slope of the tangent line:** The slope of the tangent line is equal to the Sharpe ratio of the portfolio with the highest Sharpe ratio.
* **Draw the tangent line:** The tangent line starts at the risk-free rate on the y-axis and has a slope equal to the Sharpe ratio of the portfolio with the highest Sharpe ratio. The tangent line intersects the efficient frontier at the point where the portfolio with the highest Sharpe ratio is located.

The tangent line represents the optimal portfolio for an investor who wants to maximize their risk-adjusted return. Any portfolio on the tangent line is a combination of the risk-free asset and the portfolio with the highest Sharpe ratio.